CONTROL APPLICATIONS OF THE TRYM MODEL

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Abstract: This paper outlines a control framework being developed for the TRYM model. The current procedure minimises a quadratic social loss function that targets non-steady state settings for unemployment and inflation. The iterative technique is described in detail and illustrated using the example of a temporary terms of trade shock. The effects of the shock on the model are compared for two seperate policy regimes, one using the default TRYM policy reactions and the other from the optimal control procedure. The limitations of using control techniques as an aid to the policy formation process and the benefits to be derived from implementing optimal control on a model like TRYM are breifly discussed.

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* The author is an employee of the Commonwealth Treasury. The methodology and results reported in this paper have been constructed as part of the development of the TRYM model. Helpful comments and suggestions were obtained from Peter Downes, Bruce Taplin, Chris Murphy and Andrew Johnson. All results and opinions expressed are those of the author and do not necessarily reflect those of the Treasury or the Government.

1.INTRODUCTION

This paper describes a control methodology recently developed for the TRYM model of the Australian macroeconomy. A brief outline of the model and its structure is included in section 2 of this paper. A detailed description of the TRYM model can be obtained from Taplin et.al.(1993).

The procedure involves selecting dynamic profiles for monetary and fiscal policy settings that minimise the sum of a quadratic social loss function. These "optimal" policy settings are obtained using a multivariate Gauss-Newton algorithm, analogous to that used when estimating a system of equations by multivariate non-linear least squares.

This control methodology is an adaptation of that first suggested by Fair in 1974. The fact that it is still an appealing way to control a macro model in the 1990s, suggests that the technology adopted in the early days of economic control was and still is much more reliable than the models it has been applied to.

The original framework used to build the algorithm was identical to that used in Murphy and Brennan (1993). This served as a convenient starting point for this research as the TRYM model and the Access Economics Murphy model, used by Murphy and Brennan, are of a similar size, design and are both based on the quarterly National Accounts data. Some modifications have been made to the loss function and the solution procedure to make them more applicable to the TRYM model and the simulation software used by the Treasury. The loss function selected for this work is explained in section 4 while the revised solution procedure is outlined in section 5.

Three separate policy variables are used as instruments to minimise the social loss function over a twelve and a half year time period. Monetary policy adjusts the 90 day bank bill interest rate by varying the growth rate of the money supply. Fiscal policy adjusts two instruments, representing revenue and spending. Revenue is adjusted by altering the tax rates on income from labour and capital, while expenditure is adjusted by setting the growth rate in Government Market Demand.

Explicitly separating fiscal policy into revenue and expenditure in this fashion enables the control process to take advantage of the divergent effects they have on the TRYM model, and therefore provide more realistic policy suggestions.

The procedure is demonstrated by introducing a temporary improvement to the terms of trade to the model, and outlining the optimal policy response provided by the control algorithm. The model outcomes are compared with those obtained from running the same shock under the default policy reaction functions currently used in TRYM. The default policy settings are outlined in section 3 and a familiarity with them will assist the interpretation of the simulation results outlined in section 6.

An interesting result highlighted in this work is the extent that the standard policy reaction functions determine the model's cyclical behaviour. The dynamic adjustment of the model becomes much more stable when the standard policy assumptions are replaced with a control process. The results from the shock appear to be quite stable in the control case as opposed to the more cyclical outcomes from the standard model.

The role of expectations in the model is outlined in section 2. Some of the limitations of controlling economic models, including the problem of time inconsistency and its implications to this work are discussed in section 7.

2. THE TRYM MODEL

TRYM is a macroeconometric model (as opposed to a Computable General Equilibrium model), based on quarterly time series data from the Australian National Accounts. It can be described as a medium sized model with 106 equations, 28 of which are stochastic. While this may appear large, it is relatively small compared to the most of the prominent macro models of the 1980's.

TRYM was designed to be internally consistent and to remain transparent enough for non-quantitative economists to be able to understand the source of its results.

The theoretical nature of the model can be summarised as having Keynesian short run and Neo-classical long run properties. Most of the stochastic equations in the model are specified with an identifiable long run, either as error-correction or partial adjustment equations. These long run components are used to construct an explicitly specified steady state model, that grows along a balanced steady state growth path. This steady state growth rate is supply determined and is equal to the sum of the rate of population growth and the rate of technical progress.

The model's wage equation is based around an expectations augmented Phillips curve that is vertical in the long run at a constant equilibrium unemployment rate or NAIRU (Non Accelerating Inflationary Rate of Unemployment).

The production technology in both the private and public sectors is represented by Constant Elasticity of Substitution (CES) production functions augmented by Harrod Neutral technical progress.

Expectations are modelled in TRYM in two distinct fashions. Wage earners and consumers are assumed to form expectations in a backward looking fashion, that is, they have adaptive expectations. The financial markets are assumed to have "quasi-rational" expectations. That is, agents are forward looking in some of their behaviour and adaptive in others.

They form their expectations in a forward looking manner for nominal variables, by assessing the effect of a shock on the steady state model and adjusting their expectations accordingly. (Ten year ahead expectations for the nominal exchange rate and prices are derived from the steady state model[1]). However, real long bond yields are derived from adaptive or backward looking expectations of the level of short term interest rates. Thus the financial sector of the TRYM model has a mixture of adaptive and quasi-rational expectations.

It is not assumed that agents in the financial markets know or understand the dynamic path that the model takes after a shock. However it is assumed that they anticipate the long run effects of monetary and other shocks to nominal variables.

With the current expectational structure, temporary shocks have no direct effect on either of the forward looking expectational variables in the financial sector. As a result, the outcomes from the control procedure need to be interpreted with care as the expectational set up implicitly assumes that some economic agents may make repeated mistakes in judging the course of the economy and the response of government policy.

Forward looking agents in the financial sector have particularly large incentives to eliminate any systematic expectational errors that they may be making. It would therefore be difficult to be confident about a policy's optimality, if it has been assumed that these forward looking agents do not develop an understanding of the effects of this policy.

The implications of forward looking behaviour for controlling a macro economic model, like TRYM, are elaborated upon in greater detail in section 6.

3. POLICY INSTRUMENTS

Three policy instruments are used in this paper; monetary policy, government expenditure and the tax rate. They are selected over the entire 49 quarters of the control period by the iterative framework outlined below. The settings for all three instruments are constrained to be fixed over the last 30 quarters of the control period. This is to avoid the well known “end point” problem associated with controlling economic models over finite periods of time. The end point problem occurs when a control procedure minimises the loss in the control period by stimulating the end point of the control period, pushing the subsequent losses out past the end point (sweeping the loss under the carpet).

The end point restriction works by imposing constant policy settings over the latter part of the control period. The number of quarters selected for this end point restriction depends on the dynamics of the model used, and should be long enough for the model to experience the inflationary effects of a sustained policy expansion. In this case 30 quarters is more than sufficient.

3.1 Monetary Policy

Monetary policy, is set by altering the growth rate of the money supply, which has a direct effect on the 90 day bank bill interest rate via the money demand equation[2]. This equation determines the level of the 90 day interest rate according to the ratio of nominal GNE over the volume of money, so that if nominal activity accelerates or the growth rate of the money supply decreases, the interest rate will increase.

The default monetary policy setting in TRYM, exogenously sets the money supply to grow at a rate equal to the steady state growth rate of the real economy (gr) plus the steady state inflation rate.

3.2 Tax Rate

The tax rate on labour income in the TRYM model adjusts so that the actual level of the government debt to

[1] As the model should be returning to the steady state ten years after a shock, the expectations of the nominal variables will be almost model consistent.

[2] See Taplin et.al.(1993).

GDP ratio moves towards the desired level of this ratio. The default fiscal policy setting is to extrapolate the desired level of the debt to GDP ratio as the last value experienced in history.

The tax rate on labour income is tied to the tax rate on capital income so that they remain in constant proportion to each other. This implies that any change in the tax rate on labour income is associated with a proportional increase in the tax rate on capital income. The control procedure varies this desired level of the debt to GDP ratio in order to affect the tax rates.

3.3 Government Expenditure

Government market demand is normally exogenous in a standard simulation of the TRYM model, and is extrapolated at the real steady state growth rate (gr). The control procedure selects variations around the standard growth rate of government market demand.

4. THE LOSS FUNCTION

The choice of a loss function is obviously crucial to the quality of results that are obtained from any optimal control procedure. This choice involves selecting target variables, their associated target values, the functional form of the loss from each target and providing a hierarchy of weights between the target variables. Ultimately the choice depends on the preferences of the individual controlling the model.

The loss function, used in this exercise, equation (1), includes a subjective discount rate that reduces the relative importance of loss as time proceeds. The validity of discounting the importance of welfare losses that occur in the future is a contentious issue and will not be dealt with here.

$$L=\sum_{t=1}^{49} 1\big/(1+r)^t . \left[2.56(UR_t-4)^2+0.64\left(\Delta P_t-1\frac{3}{4}\right)^2+.0036\left(\frac{Debt_t}{GDP_t}-.29\right)^2\right]$$
$$+1.69.(\Delta r90_t)^2+1.69.(\Delta Tax_t)^2+1.(Tax_t-.2)^2$$
$$+1.(\%\Delta GMD_t-gr)^2 \qquad (1)$$

where r is the inter temporal discount rate and is set to .0125 (5% p.a.)
UR is the unemployment rate,
ΔP is the through year inflation rate (GDP deflator),
$\frac{Debt_t}{GDP_t}$ is the ratio of the stock of public sector debt to of one years GDP,
r90 is the 90 day bank bill interest rate
Tax is the quarterly change in the tax rate on labour income,
GMD is government market demand, and
gr is the steady state growth rate.

The discount rate has been left at five per cent per annum, the value set by Murphy and Brennan. Their justification for this setting is that it is around the average level of the real interest rate over recent years. Results are unlikely to be particularly sensitive to this parameter unless extreme values are chosen.

Unemployment and inflation are the two primary target variables in this loss function. The weights have been set so that a one per cent deviation of unemployment from it's target will cause four times as much social loss as an equivalent deviation of inflation from it's target.

It is difficult to select subjective weighting schemes for targets like unemployment and inflation. In this case, the four to one relative difference between the two targets used by Murphy and Brennan have been maintained . However, a direct comparison is difficult as the lower target values used here cause the losses to be higher on average (or to be more precise the slope of the loss function is greater). To compensate for this effect, the absolute size of the weights on both targets has been reduced.

The non-steady state target values for unemployment and inflation imply that social loss will be experienced when the economy is in an equilibrium situation. Interestingly, this does not prevent the control procedure from providing policy that stabilise the model much more rapidly than the default policy settings.

Nor does it appear to introduce cyclical bias into the control procedure. This was tested by running the control procedure where the model was initially at its steady state. The control procedure caused unemployment and inflation to deviate only slightly from their steady state values. However, as any fall in the unemployment rate below the NAIRU leads to accelerating inflation, the deviations do not last for a significant period of time.

The equilibrium unemployment and inflation rates used in this analysis are 7 and 3 respectively[3]. In a model

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[3]The NAIRU in the TRYM model is approximately 7 per cent, the exact figure varies slightly every time the model is updated with new data. The equalibrium inflation rate is exogenous.

such as TRYM, the equilibrium level of unemployment, or NAIRU, is estimated over history as a single parameter in the wage equation and is therefore not endogenous. As a reliable explanation of the what determines the level of the NAIRU is not available, it is naively assumed to remained constant in over the control period. An interesting account of the effects on the TRYM model of a gradual reduction in the NAIRU can be found in Johnson and Downes (1994). TRYM's NAIRU estimate is comparable with other Australian models that take a similar approach to estimating their wage equations[4]. Measuring an equilibrium unemployment rate is a very difficult and contentious issue that is beyond the scope of this paper.

Regardless of the current estimate of the equilibrium unemployment rate estimated in TRYM, an unemployment rate of around seven percent would still cause significant social loss. A symmetric quadratic loss function targeting an unemployment rate of 7, suggests that an unemployment rate of 5 per cent is a worse outcome than 8 per cent, as can be seen in Figure 1. Lowering the target value has the advantage of making the loss more linear over the likely range of outcomes. It is for this reason that it was decided the symmetric specification of the loss function that incorporates the steady state targets, (SSSQ), was not appropriate.

The limitations of symmetric loss functions lead some authors to specify one sided quadratic loss functions (SSAQ). It is undoubtedly an improvement on the standard quadratic approach, but still has some problems associated with it. For example it does not provide scope for losses to be associated with very low levels of unemployment or inflation.

In a model with sticky nominal wages, a low level of positive inflation is desirable, as it facilitates downward real wage adjustments. Negative inflation may discourage borrowing and therefore suppress investment. Deflation also causes real wages to increase, retarding employment, investment and growth.

Very low levels of unemployment also have the potential to cause social loss. If unemployment reaches levels so low that employers cannot find workers with the particular skills that they require, productivity will be adversely affected. This phenomenon is known as "skills mismatch". Trivedi and Baker (1985), suggest that skills mismatch would become a problem in Australia at an unemployment rate of around 4 per cent.

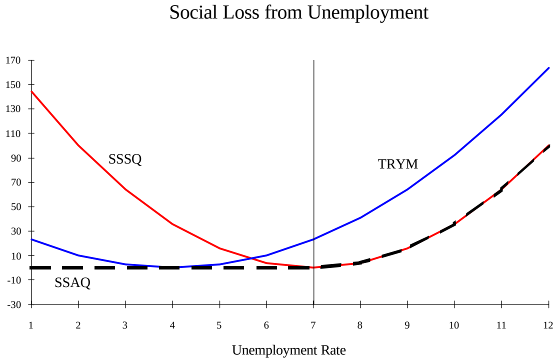

Fig. 1. SSSQ is a Steady State target Symmetric Quadratic loss,
SSAQ is Steady State target Asymmetric Quadratic loss.

In a model that has a Phillips curve specified in the wage equation, very low levels of unemployment will be associated with rapid wage inflation and vice versa. When employing an asymmetric loss function in this situation, an increase in social loss will be observed from the rapid inflation and none will be observed from very low unemployment. If the policy instruments have differing effects on the target variables, then specifying a one sided loss function may lead the control procedure to prescribe less effective policy responses.

The approach used in this paper is not without its own drawbacks. In particular, the losses caused by low levels of unemployment or inflation may not be identical to those experienced when these variables are too high. Therefore the symmetric specification is not entirely appropriate. A practical solution to these limitations will hopefully be included in future control applications to the TRYM model. One possibility is to specify a combination of a quadratic and linear loss function that would provide a steeper loss function above a target than below it.

The public sector debt to GDP ratio is the other major economic target in the loss function. It has been included to constrain fiscal policy stabilise public debt as well as respond to the other economic objectives. In the TRYM model, stabilisation of the public sector debt implies the stabilisation of the foreign debt via a standard national savings/investment framework. Therefore this term can also be interpreted as a foreign balance objective. Neither TRYM nor the control procedure are designed to elaborate upon what level of public debt to GDP may be "optimal". Therefore stabilisation is the only objective and the target value is automatically selected as the last value of the debt to GDP ratio experienced in history.

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[4] Committee on Employment Opportunities, (1993), Restoring Full Employment: A Discussion Paper, page 50.

As can be seen from equation (1), the weight put on the debt stabilisation term is much lower than that for the inflation and unemployment targets. Debt stabilisation is a medium to long term objective and a low weight allows fiscal policy the freedom to influence inflation and unemployment in the short run and obtain stabilisation in the long run. A possibly superior approach would be to have a negative intertemporal discount rate on this target, emphasising the long run nature of the objective.

The four remaining targets are included to constrain the optimisation procedure to provide more realistic policy profiles. The change in the interest rate, tax rate and growth rate of government market demand terms are included to ensure that these policy variables remain reasonably smooth.

The term targeting the level of the tax rate is included, in part, to prevent the control procedure from taking advantage of the “balanced budget multiplier” effect present in the TRYM model. This multiplier effect stimulates activity in the economy by increasing both taxes and government expenditure. This causes consumers to save less as they try to maintain their original level of consumption with their diminished after tax wages. While they are slowly adjusting their consumption behaviour, the government is spending the extra tax revenue, thus, aggregate demand is increased.

Examining balanced budget multiplier effects is a valid part of the assessment of fiscal policy. However, there are also likely to be costs involved with changing the overall level of taxation. TRYM is not designed to assess the optimal level of taxation. Hence the tax rate is targeted at its current level. This is not meant to imply a view that the current level is optimal.

The weights of all four of these targets have been adjusted to obtain results that appear to give a degree of smoothness that is credible, and policy responses that are less erratic than those experienced over the past two decades.

Although most of the discussion of optimal control often focuses on the loss function, it should be noted that the results appear to be much more sensitive to the structural trade-offs in the model and the uncertainties surrounding the impact of the policy variables, such as the effect of interest rates on the exchange rate.

5. THE CONTROL PROCEDURE

An iterative procedure is used to select the policy settings that minimise the loss function over the 49 quarter control period. An numerical approach was selected because TRYM is a non-linear model that is too large to linearise and solve the control problem analytically.

5.1 The Algorithm

The following is a brief outline of the multivariate Gauss-Newton algorithm at the core of the control process. This particular method has been chosen because it is efficient, and given our software constraints, is easy to set up. It also provides a convenient method of incorporating the "end point" restriction outlined above.

If the objective function to be minimised is expressed as

$$L = \sum l^2 ,$$

where l is a set of seven (49*1) vectors stacked on top of each other, with each of these seven vectors representing the square root of the additional loss experienced at each quarter from one of the seven target variables. The first derivative of the loss function with respect the vector of policy instruments settings, P, can then be expressed as

$$\frac{dL}{dP} = 2\sum \frac{dl}{dP}.l ,$$

where $(\frac{Dl}{DP})$ is a derivative matrix, the construction of which is set out below. The second derivative is then given by the following expression.

$$\frac{d^2L}{dPdP'} = 2\sum \left[\frac{dl}{dP}.\frac{dl}{dP'} + \frac{d^2l}{dPdP'}.l \right]$$

In a near linear model the second term inside the brackets will always be close to zero and can therefore be dropped from the expression without causing the algorithm to loose efficiency.

$$\frac{d^2L}{dPdP'} \approx 2\sum \left[\frac{dl}{dP}.\frac{dl}{dP'} \right]$$

This implies that the standard Newtonian algorithm of

$$\hat{P} = \tilde{P} - \frac{dL}{dP} \Big/ \frac{d^2L}{dPdP'} ,$$

where $\hat{P}$ is the latest update of the policy vector and $\tilde{P}$ is the policy vector from the previous iteration, becomes

$$\hat{P} = \tilde{P} - \left[\sum \frac{dl}{dP} . \frac{dl}{dP'} \right]^{-1} . \sum \frac{dl}{dP} . l \qquad (2).$$

By labelling the derivative matrix ($\frac{Dl}{DP}$), X, it highlights the fact that each successive update of the policy vector, amounts to regressing the first derivative matrix ($\frac{Dl}{DP}$) on the square root of the social loss vector (l) from the last iteration, using Ordinary Least Squares (OLS).

$$\hat{P} = \tilde{P} - \left[(X'X)^{-1} X'l \right]$$

The analogy of repeated OLS regressions can be extended to incorporate the 'end point' restriction outlined above. As the end point constraint can be expressed in the form:

$$RP = r ,$$

where R is a (147*90) matrix,
P is the vector of policy settings,
r is a (90*1) vector of zeros,

the expression for estimating equation parameters subject to a linear restriction using OLS can used update the policy vector at each iteration.

$$\hat{P} = \tilde{P} - \left\{ \breve{P} + (X'X)^{-1} R' \left[R (X'X)^{-1} R' \right]^{-1} \left(r - R\breve{P} \right) \right\} \qquad (3)$$

where $\breve{P} = (X'X)^{-1} X'l$ and
$X = \frac{dl}{dP}$

The structure of the restriction matrix, R, needed to constrain the algorithm to hold a constant policy setting for each instrument over the last thirty quarters of the control period, will not be explained in detail here. It is however extremely sparse, with only 180 non zero elements, all of which are pairs consisting of a one followed by a negative one. A full description of this restricted least squares technique can be found in most econometric text books[5].

Without the use of this constrained minimisation technique, borrowed from linear regression, the approximation used to reduce the number of simulations required to set up the derivative matrix would not be valid.

[5] Johnson, J., Econometric Methods, pages 182-186 and 204-205.

5.2 Construction of the Derivative Matrix

Starting from a baseline simulation, the square root of the loss from each target variable is calculated over an 89 quarter sample period. The separate losses for each target are then stacked into a vector, corresponding to the vector l used in section 5.1 above.

Three additional simulations are then run, each with a perturbation to one of the policy variables in the 40th quarter. Simulating the model for forty quarters before the perturbations occur, allows the effects of policy announcements on the forward looking variables to be included in the derivative matrix. It also introduces the issue of time inconsistency and forward looking behaviour into the procedure.

The vector (l) is then recalculated for each of the three perturbed simulations. The differences between the vectors are then divided by the size of the perturbation. The 40th to the 89th observations in the resulting vector represents the derivative of the loss from one target variable with respect to one policy instrument in the first quarter of the control period.

Theoretically, the derivative for each instrument should be separately calculated for every quarter in the control period. With three instruments and 49 quarters in the control period, this would involve running a total of 148 simulations, counting the initial baseline forecast, every time the algorithm is to be updated. Given that our present computer hardware and simulation software takes around fifteen minutes to conduct a simulation over forty nine quarters, this is clearly not suitable if the procedure is to be of any practical use.

As the TRYM model is almost linear in its variables, It is assumed that the derivative of the loss function with respect to a policy instrument at any point in the control period is the same as the initial derivative. This allows the algorithm to be updated using only four model simulations and does not appear to significantly effect the algorithm's efficiency.

The derivative for each target and instrument can then be arranged into 21 separate matrices. The matrix below is a (3*3) example showing how the derivative of the loss from unemployment, (UR), with respect to a change in monetary policy (M) is constructed. As the control period contains 49 observations, the dimensions of each sub-matrix is actually (49*49).

$$\frac{dUR}{dM}_t = \begin{pmatrix} \frac{dUR_1}{dM_1} & \frac{dUR_1}{dM_2} & \frac{dUR_1}{dM_3} \\ \frac{dUR_2}{dM_1} & \frac{dUR_2}{dM_2} & \frac{dUR_2}{dM_3} \\ \frac{dUR_3}{dM_1} & \frac{dUR_3}{dM_2} & \frac{dUR_3}{dM_3} \end{pmatrix}$$

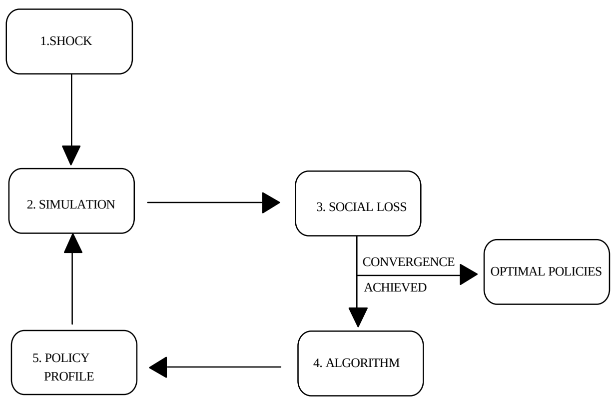
The 21 individual derivative matrices are then stacked into a larger matrix as is outlined below. This allows the simultaneous minimisation of the loss from each target, where the weightings determine the relative importance of each goal.

$$X = \begin{bmatrix} \frac{dUR}{dM} & \frac{dUR}{dT} & \frac{dUR}{dG} \\ \frac{d\Delta P}{dM} & \frac{d\Delta P}{dT} & \frac{d\Delta P}{dG} \\ \frac{dDebt}{dM} & \frac{dDebt}{dT} & \frac{dDebt}{dG} \\ \frac{d\Delta r90}{dM} & \frac{d\Delta r90}{dT} & \frac{d\Delta r90}{dG} \\ \frac{d\Delta tax}{dM} & \frac{d\Delta tax}{dT} & \frac{d\Delta tax}{dG} \\ \frac{dGmd}{dM} & \frac{dGmd}{dT} & \frac{dGmd}{dG} \\ \frac{dtax}{dM} & \frac{dtax}{dT} & \frac{dtax}{dG} \end{bmatrix}$$

Once the above matrix has been constructed it can be stored and repeatedly used in the iterative procedure. The matrix only needs to be updated if the models' structure, parameter values or the loss function are altered.

5.3 The Iterative Procedure

The flow diagram below outlines the various steps involved in the procedure.



Step 1: Impose a shock on the model.

Step 2: Conduct a standard TRYM simulation using the default policy settings.

Step 3: Use the results from this simulation are then used to evaluate the sum of the loss function, equation (1).

Step 4: Calculate the square root of the loss from each target from the most recent simulation. The resulting set of seven series are then stacked into the vector l. Obviously the order that the series are stacked into the vector must correspond to that used when constructing the derivative matrix. Equation (3) is then applied to update the most recent vector of policy settings.

Step 5: The resulting policy profiles are then extrapolated used to simulate the model again (step 2) and the procedure is repeated until convergence is achieved.

The convergence criteria employed here is rather ad hoc, it stops the procedure when the improvement in the loss function (equation (1)) from the last iteration, is less than one tenth of a per cent. Using this criteria, the algorithm normally converges after three or four iterations.

6. RESULTS

A temporary improvement in the terms of trade is used to illustrate the procedure and demonstrate the effect of the policy responses on the TRYM model. The optimal policy settings and their effects on the model are compared to those obtained using the standard policy reactions in the TRYM model.

This shock has been implemented by constructing an increase of approximately five percent in the price of commodity exports over four quarters.

In an attempt to isolate the difference between the optimal policy response and the default settings, both sets of results are expressed in terms of deviations from a forecast baseline obtained using the optimal control procedure.

The chart below depicts the deviations from baseline of the terms of trade. The difference between the two deviations in the first four quarters is due to a larger endogenous exchange rate appreciation in the control simulation. (see Figure 6).

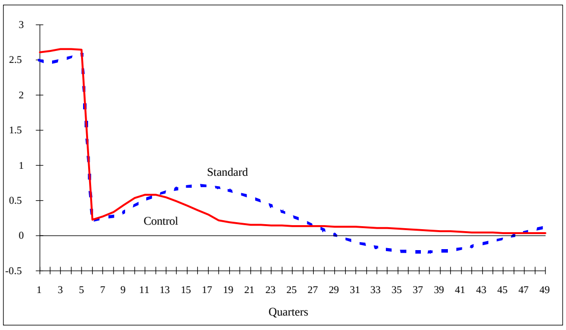


Fig 2. Terms of Trade (deviation from baseline).

The supply of commodity exports in the TRYM model is not capable of rapidly increasing in response to a price rise, this reflects the difficulty that farmers, in particular, have varying their output at short notice. In the longer term the supply of commodity exports gradually increases, as can be seen in Figure 3.

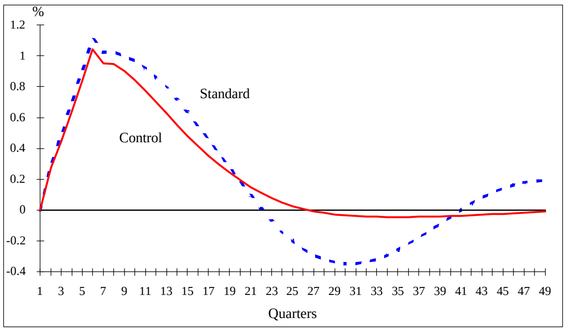


Fig 3. Volume of Commodity Exports (deviation from baseline)

The increase in both the volumes and price of commodity exports causes household incomes and wealth to increase. This quickly stimulates activity, primarily through higher consumption and business investment. This increase in activity initially causes unemployment to fall (Figure 5) and causes inflation to increase in the medium term (Figure 4). Optimal policies should be able to manage these expansionary effects and moderate the large decline in activity and the increase in inflation evident in the medium term of the standard model simulations.

As would be expected from an externally derived demand shock, the exchange rate in both simulations appreciates, insulating the domestic economy from the nominal effects of the shock. This appreciation in both simulations is caused by higher domestic interest rates and in the control case an increase in the expected future exchange rate (via the uncovered interest parity condition specified in the TRYM model).

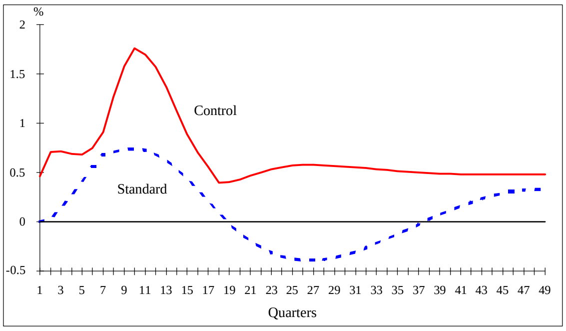


Fig 4. Trade Weighted Index (deviations from baseline).

The appreciation causes the price of imported goods and services to fall, temporarily suppressing inflationary pressures. Both simulations enjoy similar reductions in inflation over the first eight quarters, but the controlled outcome avoids most of the subsequent peak in inflation experienced in the standard simulation.

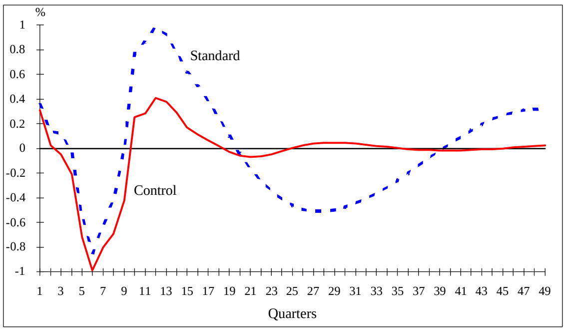


Fig 5. Inflation Rate (deviation from baseline, GDP(A) deflator, through year)

The response of the unemployment rate is depicted in Figure 6 below. The stimulatory effects of the shock increase activity and employment, causing unemployment to fall by more than 0.2 per cent in the standard simulation.

Falling unemployment and a lower price level, cause real wages to increase, creating the inflationary surge depicted above and reducing employment growth. This effect combined with a higher exchange rate, reduces competitiveness and the incentive to invest, causing a reduction in real activity.

This contraction following the positive initial effects of the shock is primarily due to the nominal rigidities and inertia in the TRYM model, and is highlighted by the hump in the unemployment graph below.

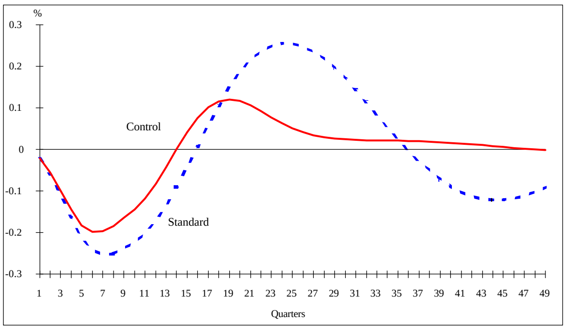


Fig 6. Unemployment Rate (percentage point deviation from baseline).

The response of the monetary and fiscal policy variables are shown below for both the standard TRYM policy reactions and the optimal control simulation.

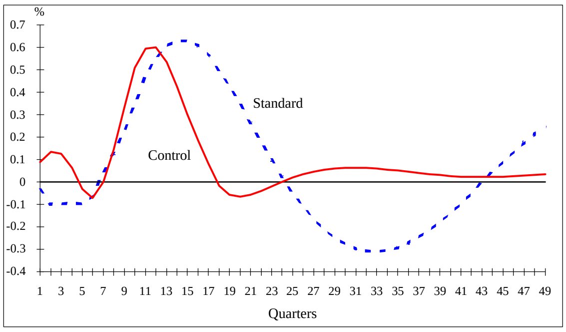
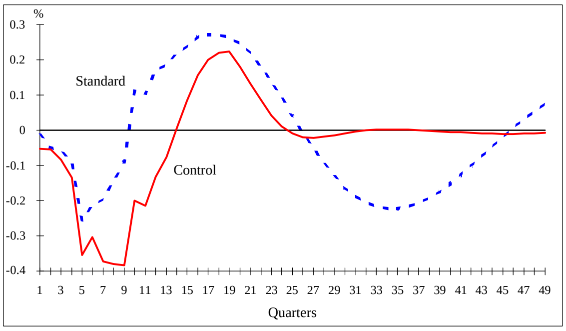


Fig 7. 90 Day Interest Rates (percentage point deviation from baseline)

While the control instrument for monetary policy is the growth rate of the money supply, in reality the stance of monetary policy is reflected by the level of the 90 day interest rate. As can be seen in Figure 7, the initial reaction of monetary policy under the standard policy reaction is to ease policy over the first seven quarters. This is in contrast to the optimal monetary policy response which increases short term interest rates by around 0.1 of a per cent for the first year. Both policy regimes then increase 90 day interest rates by around 0.6 of a percentage point over the next 18 months.

The optimal monetary policy is then eased earlier and does not fluctuate as much as is the case in the simulation using the standard TRYM monetary reaction function.



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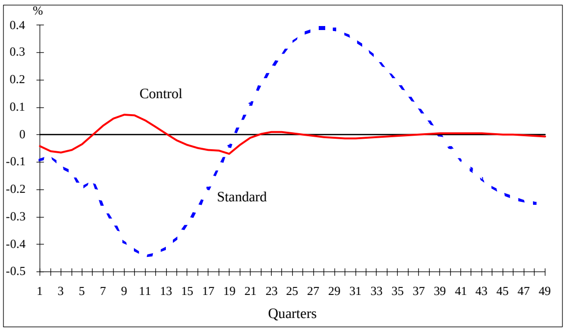
ig.8 Net Public Sector Borrowing Requirement (percentage point deviation from baseline, expressed as a proportion of nominal GDP)

The overall stance of fiscal policy can be assessed using the Net Public Sector Borrowing Requirement (NPSBR) as a proportion of nominal GDP. Figure 8 shows that both policy settings involve a contraction in fiscal policy over the first five quarters. The standard policy reaction function then loosens fiscal policy after three years, while the optimal policy setting continues to tighten policy for another twelve months. Once again the optimal fiscal policy profile is much more stable in the latter period of the simulation.

The above charts give a very brief overview of the effects of a temporary improvement in the terms of trade on the standard TRYM model and show the optimal monetary and fiscal policy settings, given the loss function specified in equation (1). As could have been expected under the control solution, both monetary and fiscal policy are initially tightened slightly, reducing the short run beneficial implications of the shock. Fiscal policy is then use to stimulate activity and alleviate the increase in unemployment while monetary policy appears to concentrate on reducing the surge in inflation. This combination of policy settings significantly reduces the negative effects of the shock experienced in the medium term of the standard simulations.



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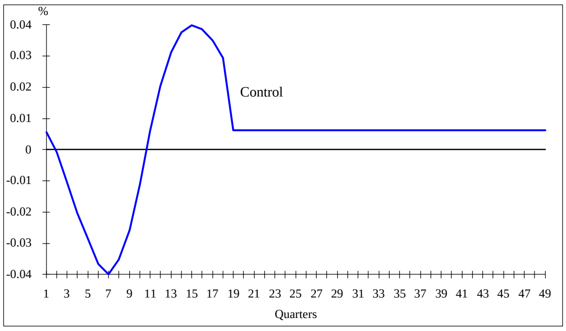
ig 9. Tax Rate on Labour Income (percentage point deviation from baseline).

The split between the two fiscal policy instruments can be seen in Figures 9 and 10. Allowing the procedure to vary the normally exogenous growth rate of government market demand, enables the tax rate on labour income to be much more stable. The implications of the variations in both instruments on the NPSBR are of a similar magnitude, with the tax rate effects being slightly larger.



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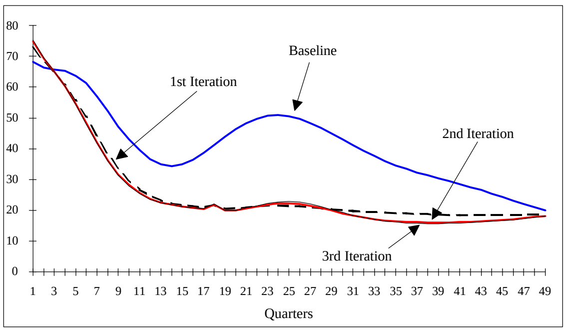
ig 10. Growth Rate of Government Market Demand (deviation from baseline)

Finally, Figure 11 below shows the loss added to the loss function at each quarter from an initial simulation and from the subsequent iterations of the control procedure required to achieve convergence. The speed of convergence and the small magnitude of the improvements from the second to the third iteration highlight the fact that TRYM is almost linear with respect to its variable. Little attention has been paid to the convergence criteria, but as can be seen in the chart below the second and third iterations are barely distinguishable from each other .



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ig 11. Level of Social Loss added at each observation in the control period.

6. BENEFITS AND LIMITATIONS

Many of the limitations of controlling an economic model are exactly the same as those affecting the more general use of macro economic models to conduct policy analysis. These common limitations include the inability of economic models to produce accurate forecasts, uncertainty regarding the correct specification of the models and uncertainty surrounding parameter and data accuracy. It is only with an appreciation of these limitations that results from controlling a macroeconomic model can be of any practical assistance to the policy formation process.

If the economic system is assumed to contain forward looking optimising agents, the problem of time inconsistency is also shared between the two endeavours. An optimal policy profile is time inconsistent if at some stage in the life of that profile, a superior outcome could be obtained by deviating from the original policy setting.

Time inconsistency is an interesting phenomenon, that is unique to systems of social behaviour and not evident in the engineering literature, from which economics originally borrowed control theory. David Currie (1985) expressed the unique implications of time inconsistency to controlling systems of social interaction as follows;

> '*As in engineering applications, the design of the control rule depends on the structure of the system to be controlled; but unlike engineering, the structure of the system depends on what control rule is perceived to be in place*".

Most authors then go on to assume that if a policy is time inconsistent it cannot be optimal, as policy makers will loose credibility and with it the effectiveness of future policy announcements. However if political, economic or institutional arrangements change then it can be difficult to maintain an announced policy path. If this is accepted, then controlling economic systems delves into the territory of game theoretic strategies.

The possibility that this procedure provides policy profiles that are time inconsistent has not yet been investigated properly. It is most probable that they are, and paying more attention to this issue is part of the future work envisaged for controlling the TRYM model.

Limitations specific to controlling macroeconomic models relate mainly to the specification of the loss function.

While all of the above limitations suggest that optimal control can not be used solely to determine macro policy settings, there are several sound reason for controlling economic models. Control results highlight the trade offs in any model of the economy and are an excellent pedagogical tools for modelling students. An effective control procedure will also reveal any 'free lunches' that may be concealed in a model's structure.

Anyone using optimal control to assist the policy formation process has to clearly define their policy objectives and the instruments they intend to use to achieve them. They will also obtain helpful, if not perfect, policy responses to different shocks. An interesting example of this in the TRYM model, is the differing policy responses suggested following equivalent demand shocks that originate from different sources. This implies that it is not only the magnitude of a demand shock that is important, but attention should also be paid to the source of the disturbance as well.

Westaway and Wren-Lewis (1990) argue that a model's overall forecasting performance can be improved by using optimal control techniques to provide more realistic policy profiles over the forecast horizon. It would not be difficult to adjust the loss function to take into account the practical factors constraining policy makers. An optimal control procedure would then provide the model with much more realistic policy profiles than the default policy reactions currently built into the TRYM model.

7. CONCLUSION

The control procedure being developed for the TRYM model of the macro economy has been described and demonstrated using the example of a terms of trade shock. A type of Gauss-Newton algorithm is employed to select the monetary and fiscal policies that minimise a quadratic loss function targeting non-steady state values of unemployment and inflation.

The specification of the loss is explained and compared to some common alternatives. Some remaining reservations are also expressed regarding the functional form of the current loss function.

Fiscal policy has been split so that expenditure and revenue can be used as two separate instruments. This utilises the differing effects and dynamic responses that both policies have in the TRYM model.

The example demonstrates that the procedure stabilises the model after the terms of trade shock and indicates the direction and size of monetary and fiscal policies that would significantly reduce the arbitrarily defined social loss from high inflation and unemployment. The initial policy reactions can be broadly described as mildly contractionary. As inflation accelerates and unemployment increases in the medium term, fiscal policy is used to reduce unemployment and while monetary policy focuses on containing inflation.

The paper then outlined some of the many limitations of obtaining policy settings from controlling macroeconomic models and outlined some areas of future work that may be carried out to try to improve this procedure.

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